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In situ measurement of the blocked force of structure-borne sound sources

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A B S T R A C T

It is shown that the blocked force of a structure-borne sound source can be obtained from measurements made *in situ*, i.e. when the source is connected to a receiver structure. This potentially removes the need for special test rigs employing blocked terminations. A corollary of this relationship is that a theoretically exact '*in situ* transfer path analysis' is possible with a fully assembled structure, such as a vehicle, without at any stage needing to separate the substructures. The results are validated by numerical simulation and measurement on beam-like sources and receivers.

1. Introduction

In order to predict structure-borne sound and vibration in assembled structures such as vehicles, ships, machinery, domestic products, buildings and many other situations there is a need to characterise vibration sources. In a few cases the required characteristics can be found by modelling, but in most cases measurement is necessary. 'Active' and 'passive' characteristics are both required in general, the latter usually being described by frequency response functions such as mobility. The focus of this paper is on the 'active' properties: the parameters commonly used are free velocity [1–4] and blocked force [5], and it has also been shown that sources can be characterised in terms of power, using the source descriptor [6], the characteristic power, the mirror power or the maximum available power [7]. The free velocity approach is the most widely accepted in principle to the extent that it has been standardised [2]. However, the standard requires that the source be operated whilst separated from any rigid support structure which is not practicable for many machines and components, particularly for running under load. Even if it is practicable to operate the source under these conditions, there is often doubt whether the operation is strictly representative. Blocked force measurements are also used, but large test rigs are required and true blocked terminations can, in practice, only be approximated over a limited frequency range [8]. In both the free velocity and blocked force approaches it can be argued that the independence of the data is achieved at the expense of realistic mounting and operating conditions since idealised free and blocked boundary conditions would never be achieved in practice.

An alternative approach is to measure operational forces at the source–receiver interface *in situ* using inverse methods [9]. Inverse methods have been widely and successfully employed, particularly in transfer path analysis (TPA) (see for example [10]) and since the measurements are partially conducted *in situ* they have the advantage of providing realistic operating and mounting conditions for the source. However, the forces obtained are not an independent property of the source being also dependent on the mobility of the receiver structure, and so are not suitable for prediction of the response of other installations. Thus, existing methods are either impractical or else they do not yield independent data. The aim of

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this paper is to develop an *in situ* measurement method which yields *independent* source characteristics, thus combining the advantages of both existing approaches.

2. Theory

In this section we develop a relationship from which the blocked forces of an active substructure can be obtained from *in situ* measurements. Consider two substructures denoted A and B as shown in Fig. 1. Substructure A acts as a source when excited by internal operating forces at location o, assumed to be inaccessible and unknown. The assembly of substructures A and B is denoted C; c represents a set of degrees of freedom on the contact interface, and b a set of degrees of freedom on substructure B, remote from the interface. $\mathbf{Y}, \mathbf{v}, \mathbf{f}$ refer to mobility, velocity and force, respectively, with lower and upper case letters representing vectors and matrices, respectively. $\mathbf{Y_{B,ca}}$ is the mobility of the separated substructure B, excited at a, with response at c. Similarly, $\mathbf{Y_{C,ca}}$ is the mobility of the assembly, excited at a, with response at c, and so on. We use the term 'generalised transfer mobility' when the excitation and response degrees of freedom do not coincide, e.g. $\mathbf{Y_{A,ca}}$, and 'generalised point mobility' when they do, e.g. $\mathbf{Y_{A,cc}}$. $\mathbf{v_b}$ is the operational velocity at b ('operational' meaning that the source is running) and $\mathbf{f_c}$ the operational force at the interface. The blocked force vector of the source (at c) is denoted $\mathbf{f_{bl}}$. Harmonic excitation is assumed throughout.

Initially, we quote the well-known relationship which forms the basis of most methods of indirect force measurement such as inverse force synthesis [9,10]. The operational forces at the interface, $\mathbf{f_c}$, are obtained from the operational velocity of the receiver structure, $\mathbf{v_b}$, by solving

$$\mathbf{v_b} = \mathbf{Y_{B,bc}} \mathbf{f_c} \tag{1}$$

A two-stage measurement is required to obtain the necessary input data. The velocity $\mathbf{v_b}$ is first measured under operational conditions. The two substructures are then separated and the generalised transfer mobility of the receiver structure, $\mathbf{Y_{B,bc}}$, measured. In practice, the reciprocity principle is often invoked and $\mathbf{Y_{B,cb}}$ is measured instead of $\mathbf{Y_{B,bc}}$ since the degrees of freedom, b, can be specifically selected for ease of applying an excitation. It is common practice to over-determine the system by adding additional responses at b, thereby improving reliability.

The operational forces can be used to evaluate the relative importance of various structure-borne sound paths, for example in transfer path analysis. However, this approach is only valid for the specific source–receiver assembly for which the operational forces are measured; knowledge of the operational forces alone is not sufficient to predict transmission in a new assembly, i.e. where the source is mounted on a new receiver. To solve this more general problem an independent characterisation of source activity is required. Therefore, in what follows we derive a new relationship in a similar form to Eq. (1) but which provides *independent* data for the source.

The operational interface forces (as obtained from Eq. (1)) can be expressed in terms of the free velocity of the source as [6]

$$\mathbf{f_c} = [\mathbf{Y_{A,cc}} + \mathbf{Y_{B,cc}}]^{-1} \mathbf{v_{fs}} \tag{2}$$

And since the free velocity and blocked force vectors are related by

$$\mathbf{v_{fs}} = \mathbf{Y_{A,cc}} \mathbf{f_{bl}} \tag{3}$$

[5] we have

$$\mathbf{f_c} = [\mathbf{Y_{A,cc}} + \mathbf{Y_{B,cc}}]^{-1} \mathbf{Y_{A,cc}} \mathbf{f_{bl}} \tag{4}$$

Substituting into Eq. (1) gives

$$\mathbf{v_b} = \{\mathbf{Y_{B,bc}} [\mathbf{Y_{A,cc}} + \mathbf{Y_{B,cc}}]^{-1} \mathbf{Y_{A,cc}}\} \mathbf{f_{bl}} \tag{5}$$

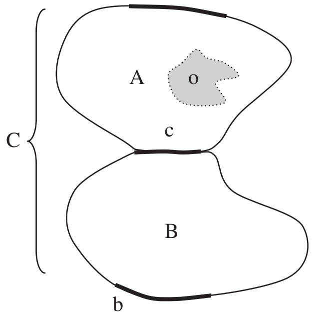

Fig. 1. Assembled structure C, comprising source substructure A and receiver substructure B: b represents reference degrees of freedom on the receiver and c represents all freedoms on the interface. o Represents the location of internal operational forces.

It will now be shown that the matrix in curly brackets is no more than the generalised transfer mobility of the coupled structure, $\mathbf{Y_{C,bc}}$. If the passive assembly is excited at c by an external force $\mathbf{f}'$, the resulting velocity at c is

$$\mathbf{v}'_\mathbf{c} = \mathbf{Y_{C,cc}}\mathbf{f}' \tag{6}$$

where the prime indicates external excitation at c. Under the same excitation the velocity at c and b in the assembly ($\mathbf{v}'_\mathbf{b}$, $\mathbf{v}'_\mathbf{c}$, respectively) are related to the interface force, $\mathbf{f}'_\mathbf{c}$, by

$$\mathbf{f}'_\mathbf{c} = \mathbf{Y}^{-1}_\mathbf{B,bc}\mathbf{v}'_\mathbf{b} = \mathbf{Y}^{-1}_\mathbf{B,cc}\mathbf{v}'_\mathbf{c} \tag{7}$$

Using Eq. (6) in Eq. (7) and rearranging we obtain

$$\mathbf{v}'_\mathbf{b} = \mathbf{Y_{B,bc}}\mathbf{Y}^{-1}_\mathbf{B,cc}\mathbf{Y_{C,cc}}\mathbf{f}' \tag{8}$$

We now use the fact that the impedance at the point of coupling for the assembly is the sum of the impedances of the substructures:

$$\mathbf{Y}^{-1}_\mathbf{C,cc} = \mathbf{Y}^{-1}_\mathbf{A,cc} + \mathbf{Y}^{-1}_\mathbf{B,cc} \tag{9}$$

which can be substituted into Eq. (8) to obtain

$$\mathbf{v}'_\mathbf{b} = \mathbf{Y_{B,bc}}\mathbf{Y}^{-1}_\mathbf{B,cc}(\mathbf{Y}^{-1}_\mathbf{A,cc} + \mathbf{Y}^{-1}_\mathbf{B,cc})^{-1}\mathbf{f}' = [\mathbf{Y_{B,bc}}(\mathbf{Y_{A,cc}} + \mathbf{Y_{B,cc}})^{-1}\mathbf{Y_{A,cc}}]\mathbf{f}' \tag{10}$$

The matrix in square brackets on the right-hand side of Eq. (10) is recognised as the same as that appearing in Eq. (5), and since it is the matrix that transforms the vector of forces applied at c to the velocity at b it is equal to the generalised transfer mobility matrix of the coupled structure. Thus, we have

$$\mathbf{Y_{B,bc}}[\mathbf{Y_{A,cc}} + \mathbf{Y_{B,cc}}]^{-1}\mathbf{Y_{A,cc}} = \mathbf{Y_{C,bc}} = \mathbf{Y}^{T}_\mathbf{C,cb} \tag{11}$$

and substituting into Eq. (5) gives

$$\mathbf{v_b} = \mathbf{Y}^{T}_\mathbf{C,cb}\mathbf{f_{bl}} \tag{12}$$

Eq. (12) is the main result of this section and has profound implications. It was reported in [11,12]. It should also be noted that the expression for the 'synthesised' force of a single degree of freedom system derived in [13] is in fact a special case of Eq. (12).

The left-hand side of Eq. (12) is the operational velocity measured at arbitrary points on the receiver structure and is the same velocity as would be used in conventional inverse force synthesis or TPA. The first term on the right-hand side is the generalised transfer mobility relating degrees of freedom at the interface to those on the receiver structure. These degrees of freedom are the same as used in conventional methods; the important difference is that the mobilities here are of the *coupled* structure rather than, as per the usual inverse methods, substructure B. The second term on the right-hand side is the blocked force vector. Thus, by solving Eq. (12) the activity of the source can be independently characterised by measurements *in situ*. There is no need to separate the two substructures. Additionally, existing techniques for improving the reliability of the solution to Eq. (1) are equally applicable for solving Eq. (12) since the problems are of similar form.

3. Numerical simulation: coupled beams

In this section Eq. (12) is tested by numerical simulation of a beam source and receiver connected end-to-end as shown in Fig. 2. This arrangement is representative of structure-borne sound sources in pipes, such as in-line pumps and valves and has the advantage there are closed-form analytical solutions available for both the source beam and the coupled structure [14]. The properties of both beams were computed for typical steel, 8 mm thick × 25 mm wide, the source and receiver beams were 0.6 and 1.0 m long, respectively. Internal 'operating' forces were simulated by applying a unit force f_o

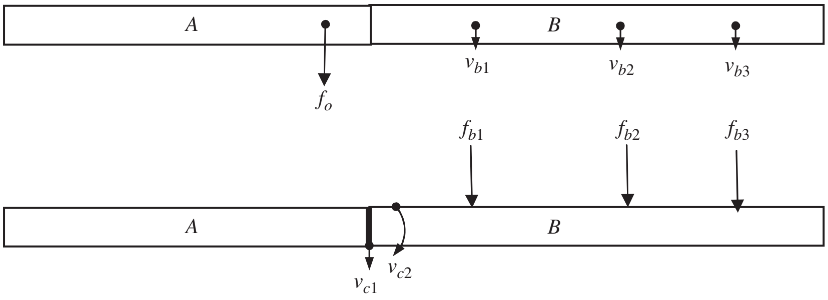

Fig. 2. Free–free beams used for numerical simulations: (a) operational velocities on receiver $\mathbf{v_b}$ and (b) generalised transfer mobility of the assembled structure $\mathbf{Y_{C,cb}}$.

at a point on the source beam. Note that, in general, a force and a moment act at the interface c, so two reference points on the receiver are theoretically sufficient to resolve the blocked force vector. However, we define three points in order to include the possibility of over-determining the system.

The exact blocked force and moment were obtained by calculating the free velocity and mobility matrix of the free source beam and then applying Eq. (6). Results are shown in Fig. 3. When exact input data is used, the blocked force and moment are predicted exactly using Eq. (12) (curve not shown). Therefore, in order to replicate some measurement errors we have simulated the finite difference approximation which would normally be used in a real measurement to obtain the angular velocities from the signals of two closely spaced accelerometers (a separation of 10 mm has been assumed between response positions). Results, shown in Fig. 3, indicate relatively small errors in the predicted blocked force and moment at low frequencies which increase at higher frequencies due to the increased finite difference error in the input data. As with any inverse problem, there is an amplification of the input errors due to the matrix inversion. The third curve in Fig. 3 indicates that the error is almost completely removed by using a third reference point on the receiver so as to over-determine the system. No regularisation or optimisation has been used in this simulation and the results could undoubtedly be improved further. However, it is of more interest to test the procedure in a real measurement, where there may be additional sources of error, and such a validation is presented in the following section.

4. Validation by measurement

In this section, Eq. (12) is applied using measurements from a beam source attached to one of two receiver beams in laboratory tests (see also [11,12,15]). The arrangement is illustrated in Fig. 4, all beams being of steel. The source beam measured 6.5 mm × 3 cm × 30 cm with 6.5 mm × 3 cm × 3 cm feet. The first receiver measured 9.5 mm × 3.8 cm × 59.5 cm and the second was nominally identical except for the length of 49.2 cm. The differing lengths of the receiver beams meant that their resonant frequencies, and therefore also their mobilities, were significantly different. Unlike the previous numerical simulation, the reference degrees of freedom on the receiver were selected so as to coincide with the contact degrees of freedom, i.e. a translation and a rotation at each of the two contact points making four degrees of freedom in total. Internal operating forces were represented by tapping the source beam with a force hammer at an ‘internal’ point. All ‘operational’ measurements were then normalised to the input force so as to avoid any variation in source activity.

Blocked forces and moments were obtained by a two stage measurement procedure: in the first the ‘operational’ velocity and angular velocity were measured at the reference locations on the contact interface and in the second, the mobility matrix of the coupled structure was measured at the same points. Eq. (12) was then used to solve for the blocked forces. Forces and moments were accounted for at each contact point giving a 4 × 4 mobility matrix. Moment mobilities were measured by a finite difference method described in [16,17].

The question of how to validate the blocked force requires careful thought since, unlike the numerical simulation, the true value cannot be obtained directly. The approach adopted was to use the measured free velocity of the source as a reliable reference; the blocked force was then validated by employing Eq. (3) and comparing the resulting calculated free velocity with the directly measured value.

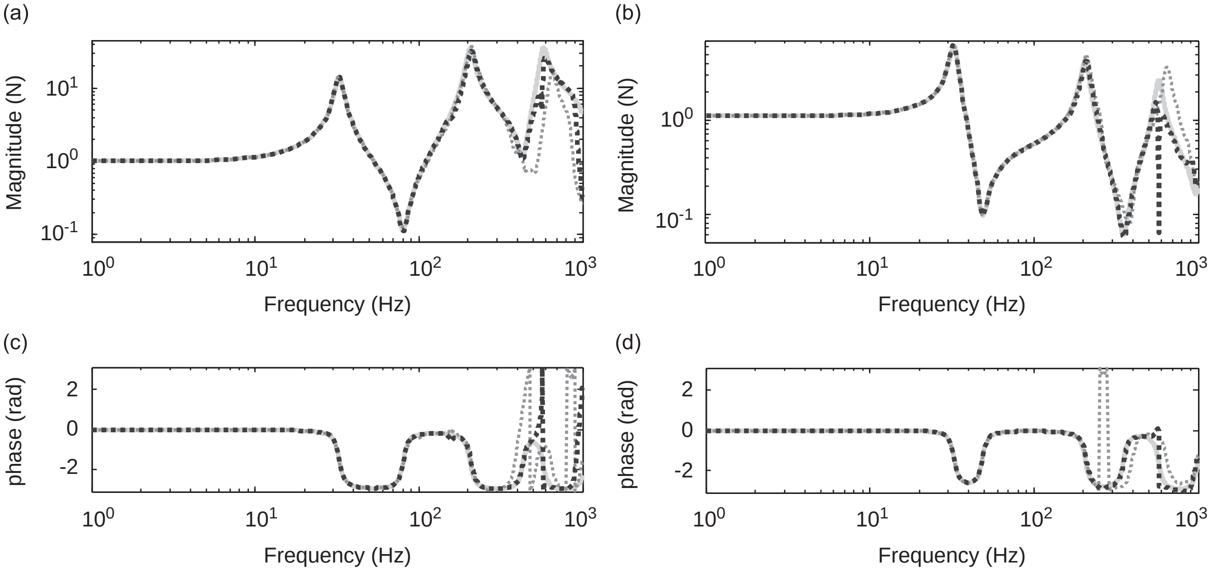

Fig. 3. (a) Blocked force magnitude and (c) phase, (b) blocked moment magnitude, (d) phase: —— exact; - - - - - - Eq. (12) with input errors, determined; - - - - - Eq. (12) with input errors, over-determined.

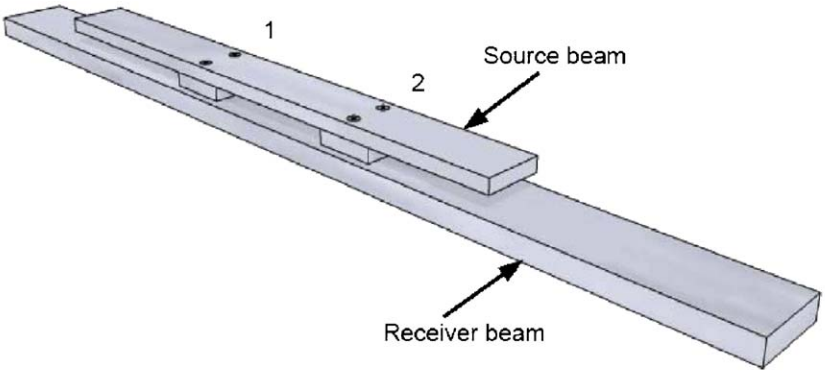

Fig. 4. Coupled source and receiver beams used for measurement of blocked forces *in situ*. Source beam fitted with 2 × 3cm^2 footings rigidly attached to receiver beam by screws at both contacts.

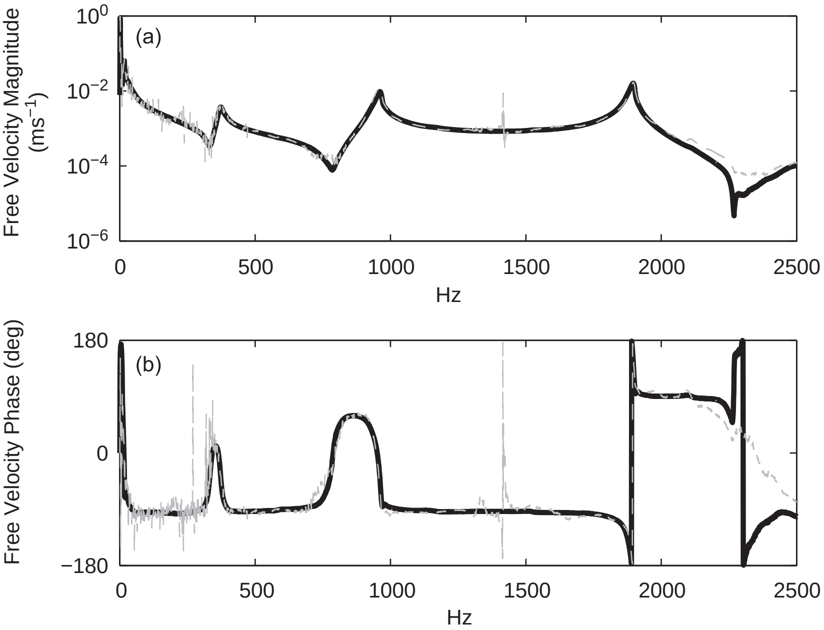

Fig. 5. (a) Free velocity magnitude and (b) phase of the source beam: —— directly measured; ----- calculated using blocked force measured *in situ*.

Results are shown in Fig. 5. A single degree of freedom, the translational free velocity at point 1, is plotted but note that force and moment contributions from both contact points had to be accounted for (in Eqs. (3) and (12)) in order to achieve this result. An excellent agreement is seen between the measured and calculated free velocity. This validates the *in situ* method for measurement of blocked force using Eq. (12). Furthermore, it provides an indication of the validity of the 'black box' approach to source characterisation which is often employed but rarely verified. Fig. 6 illustrates the difference between operational forces and blocked forces. The former are shown in the left plot (point 1, translational direction) for the same source on two different receivers. A significant variation is seen between the two curves, indicating that the operational force is strongly dependent on the receiver mobility. On the other hand, the right hand plot shows close agreement between the two estimates of blocked force obtained from *in situ* measurements on the two receivers. This shows the extent to which the *in situ* method is effective at removing the influence of the receiver. (The spectrum of blocked force is relatively flat because we have represented internal operating forces by a unit force from the hammer as described above.) Moreover, Fig. 6 illustrates that the blocked forces are independent of the receiver mobility, i.e. they are an independent property of the source, at least for this idealised case. This finding may or may not extend to practical vibration sources since it is feasible that some source mechanisms are dependent on the constraints presented by the receiver structure. However, if there were to be such a dependence this would not invalidate the *in situ* approach; on the contrary, it would emphasise the importance of making measurements on a representative receiver structure for which the *in situ* approach is ideally suited.

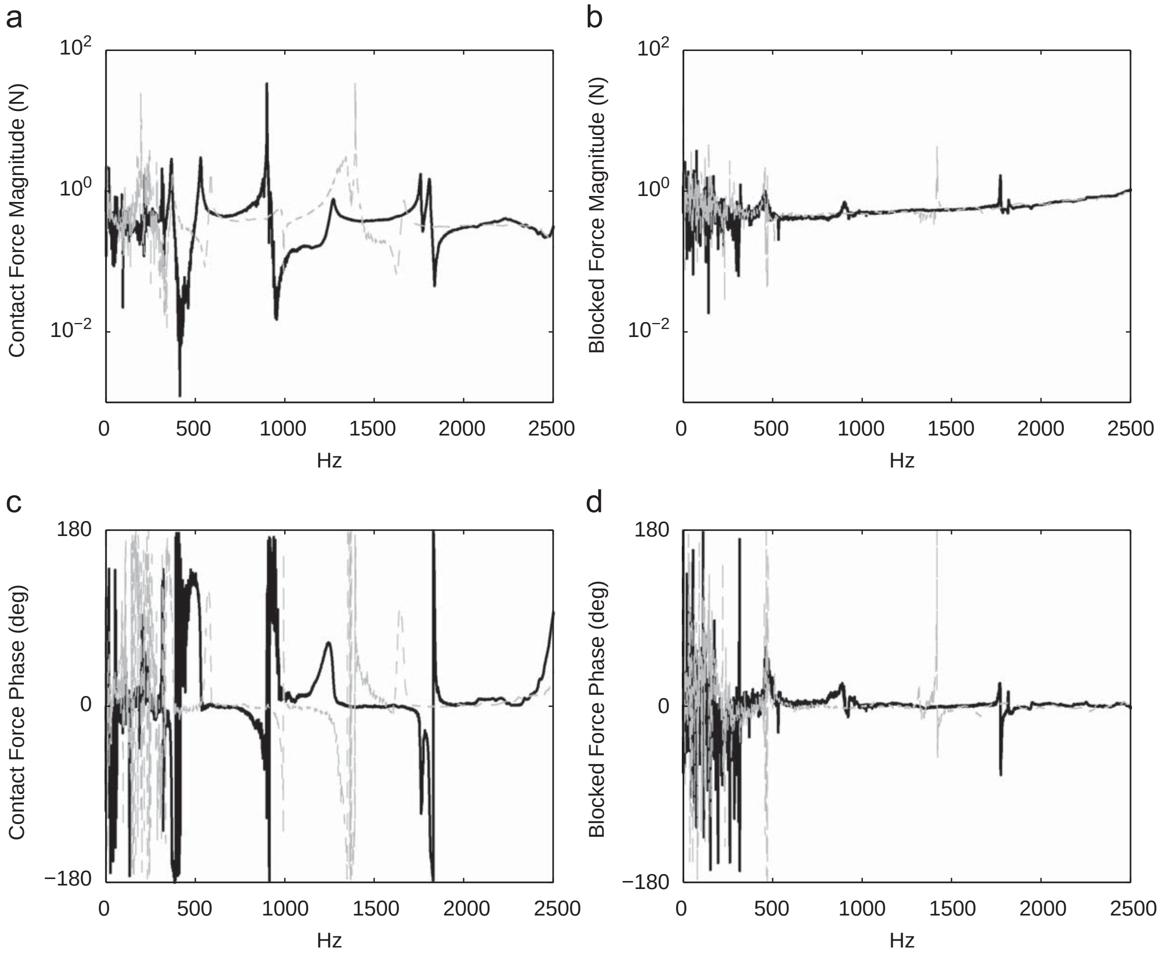

Fig. 6. (a) Operational force magnitude and (c) phase, (b) blocked force magnitude and (d) phase, obtained from measurement for a source beam on two receiver beams: —— measured on the source and receiver system shown in Fig. 4; - - - - - the same source on the second receiver beam.

As this was an ideal experiment, additional errors in input data would be expected in practice, some of which would be dictated by the properties of the receiver structure. On the other hand various techniques exist which could be employed to improve the reliability of the results (see for example [18,19]) and it is reasonable to expect to be able to achieve comparable accuracy to that obtained in inverse force synthesis and TPA.

5. Discussion

In TPA, the forces obtained by solution of Eq. (1) are used to calculate the velocity or sound pressure at remote points of the assembly. This procedure can be simplified considerably by using the new blocked force relationship: Eq. (12) can be generalised to

$$\mathbf{r}_{\mathbf{C},\mathbf{ro}} = \mathbf{H}^{T}_{\mathbf{C},\mathbf{cr}}\mathbf{f}_{\mathbf{bl}} \tag{13}$$

where $\mathbf{r}_{\mathbf{C},\mathbf{ro}}$ is any response vector at a general set of locations r and $\mathbf{H}^{T}_{\mathbf{C},\mathbf{cr}}$ is the corresponding transfer function. Importantly, we note that this transfer function is for the *assembled* structure rather than, as per conventional TPA, substructure B. (See [8] where this relationship is also given.) Thus, by using Eq. (12) in an inverse sense to obtain blocked force and Eq. (13) in a forward sense to predict responses, a TPA can be conducted without the need to separate the substructures. This could considerably speed up the TPA process. A second important advantage is that, unlike the operational forces obtained in conventional TPA, the blocked forces provide an independent characterisation of the source which can be transferred to other assemblies incorporating the same source, and can even potentially be used as input to numerical models such as finite element models.

6. Conclusions

The main result in this paper is that the blocked force of a source can be obtained from *in situ* measurements on coupled structures. There are two profound implications. First, it proves that the active properties of vibration sources can be found from measurements in the installed condition, i.e. without the need for special test rigs in which either blocked or free conditions are approximated. Secondly, there are significant advantages for transfer path analysis in that a theoretically exact '*in situ* TPA' is possible without any need to separate the source and receiver sub-structures, and with the additional advantage that the source data is transferrable to other installations and may potentially serve as input to numerical models such as finite element models.

The required measurements and analysis are of a similar form to those used in conventional inverse force synthesis and TPA, so the same well-developed techniques, such as regularisation, over-determination and use of principal component analysis, can potentially be employed to improve the quality of the solution. As with inverse force synthesis techniques, the operational responses can be measured at arbitrary reference locations on the receiver so there is no need to access the source–receiver interface during operation.

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References

[1] T. Ten Wolde, G.R. Gadefelt, Development of standard measurement methods for structure-borne sound emission, *Noise Control Engineering Journal* 28 (1987) 5–14.
[2] International Standards Organisation ISO 9611, Acoustics—characterisation of sources of structure-borne sound with respect to sound radiation from connected structures—measurement of velocity at the contact points of machinery when resiliently mounted, 1996.
[3] L. Cremer, M. Heckl, B.A.T. Petersson, *Structure Borne Sound*, Springer, Berlin, 2005.
[4] R.A. Fulford, B.M. Gibbs, Structure-borne sound power and source characterization in multi-point-connected systems, part 2: about mobility functions and free velocities, *Journal of Sound and Vibration* 220 (2) (1999) 203–224.
[5] P. Gardonio, M.J. Brennan, Mobility and impedance methods in structural dynamics, in: F. Fahy, J. Walker (Eds.), *Advanced Applications in Acoustics, Noise and Vibration*, London, 2004, pp. 389–447.
[6] J.M. Mondot, B. Petersson, Characterization of structure-borne sound sources: the source descriptor and the coupling function, *Journal of Sound and Vibration* 114 (3) (1987) 507–518.
[7] A.T. Moorhouse, On the characteristic power of structure borne sound sources, *Journal of Sound and Vibration* 248 (3) (2001) 441–459.
[8] Gaudin, L. Gagliardini, Recent improvements in road noise control, *SAE 2007 Noise and Vibration Conference and Exhibition*, SAE International, 2007.
[9] J. Dobson, E. Rider, A review of the indirect calculation of excitation forces from measured structural response data, *Proceedings of the Institution of Mechanical Engineers* 204 (1990) 69–75.
[10] D. Vandenbroeck, W.S.F. Hendricx, Interior road noise optimization in a multiple input environment, *Institution of Mechanical Engineers Conference Publications* 3 (1994) 53–58.
[11] A.S. Elliott, A.T. Moorhouse, G. Pavić, Characterisation of a structure borne sound source using independent and in-situ measurement, *19th International Congress on Acoustics*, Madrid, 2007.
[12] A.S. Elliott, A.T. Moorhouse, Characterisation of structure borne sound sources from measurement in-situ, *Proceedings Acoustics 2008*, Paris, 2008.
[13] H.-Y. Lai. Alternative test methods for measuring structure-borne sound power, *Inter-Noise 2006*, Honolulu, 2006.
[14] J.X. Su, Simplified Characterisation of Structure-Borne Sound Sources with Multi-Point Connections, Ph.D. Thesis, University of Liverpool, Liverpool, UK, 2003.
[15] A.S. Elliott, Characterisation of Structure Borne Sound Sources In-situ, Ph.D. Thesis, University of Salford, Greater Manchester, UK, 2009.
[16] S.S. Sattinger, A method for experimentally determining rotational mobilities of structures, *Shock and Vibration Bulletin* 50 (2) (1980) 17–28.
[17] A.S. Elliott, G. Pavić, A.T. Moorhouse, Measurement of force and moment mobilities using a finite difference technique, *Proceedings Acoustics 2008*, Paris, 2008.
[18] M. Blau, Indirect measurement of multiple excitation force spectra by FRF matrix inversion: influence of errors in statistical estimates of FRFs and response spectra, *Acoustica* 85 (1999) 464–479.
[19] A.N. Thite, D.J. Thompson, The quantification of structure-borne transmission paths by inverse methods. Part 1: improved singular value rejection methods, *Journal of Sound and Vibration* 264 (2003) 411–431.